Rating rules / Puzzles. Ordering the rules

Goto page Previous 1, 2, 3 ... , 26, 27, 28 Next

newtopic postreply Sudoku Players' Forums Forum Index -> Advanced solving techniques

View previous topic :: View next topic

Author	Message
denis_berthier	Posted: Wed Sep 16, 2009 9:21 am Post subject: quote

Joined: 19 Jun 2007
Posts: 807
Location: Paris, France

eleven,
thanks.
I didn't have time today but I'll run it. I want to see how the SE varies with the number of clues.

Back to top profile pm www

denis_berthier Posted: Fri Sep 18, 2009 9:31 pm Post subject: quote

Joined: 19 Jun 2007
Posts: 807
Location: Paris, France

THE VERY SURPRISING BEHAVIOUR OF A PURE BOTTOM-UP GENERATOR

After I introduced the (top-down) controlled-bias generator and eleven implemented it as a modification of top-down suexg, he also introduced a similar modification in bottom-up suexg (suexg1.4) - which results in what could be called a pure bottom-up generator.
[Exercise: why can't we apply here my theory of the (top-down) controlled-bias generator and define a priori correction factors?
Notice that this is a purely academic question, because the coefficients would have to be so large that it would make the results very unstable.]

Eleven noticed an unexpectedly small average number of clues:

eleven wrote:

> One more puzzling result. I generated minimal puzzles bottom up without dropping any clues, i.e. i only took unique puzzles, if they were minmal from the adding clue phase.
> Maybe Mike can repeat it to exclude coding errors.
>
> **Code:**
>
> ```
> 20:      3
> 21:     46
> 22:    524
> 23:   2301
> ```

```
24:   3757
25:   2549
26:    710
27:    106
10000 puzzles, av. clue number 24.0804
```

To my surprise the average clue number only is slightly higher and still clearly under the value from top down generation.

These results were confirmed by Mike, using his own generator - which almost certainly excludes the idea of a bug.

Still more surprising is the mean value of the SER for any fixed number of clues: it is significantly smaller than for any other random collection I've ever seen.
I've used eleven's algorithm to generate 100,000 puzzles (with seed 0). It took ~ two CPU days (much slower than standard suexg, but very much faster than controlled-bias suexg-cb).
Here are the results (number of clues, number of puzzles, mean value, standard deviation):

Code:

```
#Clues #Puzzles      E(SER)      s(SER)
19     0*            0           0.0
20     14*           2.529       0.92
21     492           2.829       1.93
22     5703          2.924       1.20
23     23872         3.030       2.09
24     39021         3.208       2.21
25     25818         3.492       2.35
26     7388          3.832       2.45
27     943           4.279       2.50
28     45            4.882       2.65
29     3*            5.633       2.15
30     0*            0           0.0
31     0*            0           0.0
mean   100000        3.276       2.24

*: too few puzzles in the sample, unreliable results
on the line
```

#clues: mean = 24.07, sd = 1.06

As we've been used to it, we have a trend more clues => higher complexity.
But, for each number of clues, compare the SER values with those for the top-down and bottom-up versions of suexg - see my web pages for them.
(Due to the sub-sample sizes, only values in the range 21-28 clues are meaningful, but this is enough to be surprised.)
And this definitely contradicts the idea that the number of clues was the only parameter of complexity.

Notice that my current computations with the controlled bias generator (still in a preliminary stage) lead to a similar general conclusion, although with different details.

Back to top profile pm www

eleven

Joined: 10 Feb 2008
Posts: 474

Posted: Sat Sep 19, 2009 4:14 am Post subject: quote

You reminded me on an open question, i still cant answer.

My naive approach was, that an algorithm

Code:

```
0. Start with an empty grid
1. Randomly select an empty cell and a number and add
it
2. If there are multi sulutions, repeat step 1,
   if the puzzle is minimal, report it,
   restart at step 0 (we have no solution or a unique
solution)
```

would find (valid and) minimal puzzles with n clues with a probability of 1/(9^n * choose (81,n)).
Then we could get a real distribution by means of p(n+1)/p(n) = 9*(81-n)/(n+1).

Now this is all but efficient, and we change step 2 to

Code:

```
2. If there are multi sulutions, repeat step 1,
   if the puzzle is minimal, report it and restart at
step 0,
   if it is unique, restart at step 0,
   if it has multi solutions, repeat step1,
   if it has no solution, remove the last number and
repeat step 1
```

This guarantees, that each try will at least end up with a unique puzzle.
Thus when adding clue k, we dont choose one of 9*(81-k) possibilities, but only one of the possibilities, which leave a valid solution. So if pk is the probability, that an additional clue leaves a valid puzzle, each (valid and) minimal puzzle would be found with a probability of p1*p2*...*pn.
But this obviously is not true.
p1=1, p2=700/720 and the rest i only could calculate approximately. In 10000 tries i got this for 19-29:

Code:

```
      p(valid, when adding a number)
 19:      0.52149
 20:      0.48931
 21:      0.45513
 22:      0.41765
 23:      0.37584
 24:      0.32904
 25:      0.28219
 26:      0.23968
 27:      0.20395
 28:      0.17886
 29:      0.16070
```

The fact, that after finding an unique n-clue puzzle we dont search for an (n+1)clue, does not change much for me for less 27-clues. because it only marginally reduces the number of tries for higher clue puzzles, as the following sample shows:

Code:

```
2622707 tries, 10000 minimals
       tries     uniq   min
19:    2622707      0      0
20:    2622707      0      0
21:    2622707     42     37
22:    2622665    825    555
23:    2621840   5739   2308
24:    2616101  23806   3888
25:    2592295  63461   2430
26:    2528834 120371    691
27:    2408463 176261     85
28:    2232202 217055      6
29:    2015147 234934      0
```

So what am i blind for and - probably the harder question - why should this method prefer easier puzzles for a fixed number of clues ?

Back to top

profile pm

denis_berthier

Joined: 19 Jun 2007
Posts: 807
Location: Paris, France

Posted: Sat Sep 19, 2009 6:18 am Post subject:

quote

eleven wrote:

> My naive approach was, that an algorithm
>
> **Code:**
>
> ```
> 0. Start with an empty grid
> 1. Randomly select an empty cell and a number
> and add it
> 2. If there are multi sulutions, repeat step 1,
>    if the puzzle is minimal, report it,
>    restart at step 0 (we have no solution or a
> unique solution)
> ```
>
> would find (valid and) minimal puzzles with n clues with a probability of 1/(9^n * choose (81,n)).
> Then we could get a real distribution by means of p(n+1)/p(n) = 9*(81-n)/(n+1).

But this is not true, because the number of paths from an empty grid upwards to a minimal puzzle is not the same for all the puzzles at level n.
(The trick I had to use to circumvent this problem for the top-down approach - indexing the puzzles with the solution grids - doesn't work here.)

Back to top

profile pm www

eleven

Joined: 10 Feb 2008
Posts: 474

Posted: Sat Sep 19, 2009 7:15 am Post subject:

quote

denis_berthier wrote:

> But this is not true, because the number of paths from an empty grid upwards to a minimal puzzle is not the same for all the puzzles at level n.

I see, that the ratio is not correct because of the invalid puzzles with k<=n clues. But it must have to do with some clustering (non uniformly distribution) of minimal puzzles, that [edit] the ratio is not correct for the second algorithm and the probabiltity to find one in the same level is not equal for all, doesn't it ?

Back to top

denis_berthier Posted: Sat Sep 19, 2009 8:12 am Post subject:

Joined: 19 Jun 2007
Posts: 807
Location: Paris, France

> **eleven wrote:**
>
> > **denis_berthier wrote:**
> > But this is not true, because the number of paths from an empty grid upwards to a minimal puzzle is not the same for all the puzzles at level n.
>
> I see, that the ratio is not correct because of the invalid puzzles with k<=n clues.
> But it must have to do with some clustering (non uniformly distribution) of minimal puzzles, that [edit] the ratio is not correct for the second algorithm and the probabiltity to find one in the same level is not equal for all, doesn't it ?

The second algorithm is really complex, probably very biased and I don't know how to analyse it formally.

Back to top

eleven Posted: Sat Sep 19, 2009 8:29 am Post subject:

Joined: 10 Feb 2008
Posts: 474

Thats what i like. 5 lines of simple code (beside of the solver) and we cant say what comes out

Maybe we can find simplified graphs, which characterize the problem (2x2 sudoku ?).
My conjecture is, that puzzles in clusters are found more often - and that they are easier on average. But you know that my sudoku conjectures always are wrong

Back to top

denis_berthier Posted: Sat Sep 19, 2009 8:53 am Post subject:

Joined: 19 Jun 2007
Posts: 807
Location: Paris, France

> **eleven wrote:**
>
> 5 lines of simple code (beside of the solver) and we cant say what comes out

Or a GOTO 1 instead of a GOTO 2 and we get controlled bias instead of top-down.
Or E=mc^3 instead of E=mc^2 and we would already have blown the earth to pieces.

> **eleven wrote:**

> Maybe we can find simplified graphs, which characterize the problem (2x2 sudoku ?).
> My conjecture is, that puzzles in clusters are found more often - and that they are easier on average.

The question is, is it worth investigating the bottom-up approach, which is known to be more biased than the top-down? What exactly do we expect of it?
I'd like to understand my above results about your modified version of bottom-up, i.e. why it leads to simpler puzzles for every n, but I have another puzzling question with controlled-bias (although I need more puzzles before I can be sure).

Back to top profile pm www

eleven

Joined: 10 Feb 2008
Posts: 474

Posted: Sun Sep 20, 2009 2:34 am Post subject: quote

denis_berthier wrote:

> The question is, is it worth investigating the bottom-up approach, which is known to be more biased than the top-down? What exactly do we expect of it?

Its just personal interest. Both you and Red Ed only stated, that it must be biased, because the distribution is not correct. It also was not proven, that the bias cant be controlled by some constants.

Here is a proof:
Take a "sudoku" with 2 cells and 3 numbers and the "minimals" 13, 21, 31.
If cell 1 is selected first, all minimals are found with probability 1/3.
But if cell 2 is selected first, p(13)=1/2, while p(21) and p(31) are 1/4.

This also confirms my conjecture, that my sudoku conjectures always are wrong The example shows that the algorithm tends to NOT to find the clustered minimals.

Back to top profile pm

denis_berthier

Joined: 19 Jun 2007
Posts: 807
Location: Paris, France

Posted: Sun Sep 20, 2009 8:42 am Post subject: quote

eleven wrote:

> Both you and Red Ed only stated, that it must be biased, because the distribution is not correct. It also was not proven, that the bias cant be controlled by some constants.

Proving that something can't be done is often very difficult.
But what's clear for this full bottom-up generator is that there are no obvious relations P(n+1)/P(n) as there were for the top-down controlled-bias.

Back to top profile pm www

Red Ed

Posted: Sun Sep 20, 2009 8:59 am Post subject: quote

denis_berthier wrote:

Joined: 06 Jun 2005
Posts: 712

> **eleven wrote:**
>
> > My naive approach was, that an algorithm
> >
> > **Code:**
> > ```
> > 0. Start with an empty grid
> > 1. Randomly select an empty cell and a
> > number and add it
> > 2. If there are multi sulutions, repeat
> > step 1,
> >    if the puzzle is minimal, report it,
> >    restart at step 0 (we have no solution
> > or a unique solution)
> > ```
> > would find (valid and) minimal puzzles with n clues with a probability of 1/(9^n * choose (81,n)).
> > Then we could get a real distribution by means of p(n+1)/p(n) = 9*(81-n)/(n+1).
>
> But this is not true, because the number of paths from an empty grid upwards to a minimal puzzle is not the same for all the puzzles at level n.
> (The trick I had to use to circumvent this problem for the top-down approach - indexing the puzzles with the solution grids - doesn't work here.)

Actually *eleven* is correct with his formula, though as he points out it is a totally impractical algorithm. Only his modified algorithm, the description of which admittedly I don't fully understand, appears irredeemably biased.

Back to top profile pm

denis_berthier

Joined: 19 Jun 2007
Posts: 807
Location: Paris, France

Posted: Sun Sep 20, 2009 9:04 am   Post subject: quote

> **Red Ed wrote:**
>
> > **denis_berthier wrote:**
> >
> > > **eleven wrote:**
> > >
> > > > My naive approach was, that an algorithm
> > > >
> > > > **Code:**
> > > > ```
> > > > 0. Start with an empty grid
> > > > 1. Randomly select an empty cell and
> > > > a number and add it
> > > > 2. If there are multi sulutions,
> > > > repeat step 1,
> > > >    if the puzzle is minimal, report
> > > > it,
> > > >    restart at step 0 (we have no
> > > > solution or a unique solution)
> > > > ```
> > > > would find (valid and) minimal puzzles with n clues with a probability of 1/(9^n * choose (81,n)).
> > > > Then we could get a real distribution by means of p(n+1)/p(n) = 9*(81-n)/(n+1).
> > >
> > > But this is not true, because the number of paths from an empty grid upwards to a minimal puzzle is not the same for all

> > the puzzles at level n.
> > (The trick I had to use to circumvent this problem for the top-down approach - indexing the puzzles with the solution grids - doesn't work here.)
>
> Actually *eleven* is correct with his formula.

The formula isn't correct, unless you count the no-solution puzzles.

Back to top

Red Ed

Joined: 06 Jun 2005
Posts: 712

Posted: Sun Sep 20, 2009 9:20 am Post subject:

Actually yes I was counting non-minimal puzzles (i.e. no output) in step 3; so in that sense his first formula's right but then his "real distribution" formula would need to be changed to keep track of the number of trials. OK, pointless diversion, it's a fair cop.

EDIT: corrected previous nonsense re distribution estimation

Last edited by Red Ed on Sun Sep 20, 2009 10:01 am; edited 1 time in total

Back to top

eleven

Joined: 10 Feb 2008
Posts: 474

Posted: Sun Sep 20, 2009 9:44 am Post subject:

Red Ed wrote:

> Actually *eleven* is correct with his formula, though as he points out it is a totally impractical algorithm.

Thanks for confirmimg that.

Quote:

> Only his modified algorithm, the description of which admittedly I don't fully understand, appears irredeemably biased.

I thought the modified algorithm (which is the one, suexg uses), is clear. You are adding clues, as long as the puzzle has multi solutions. If adding a (randomly selected) clue would lead to an invalid puzzle with 0 solutions, you remove it again and try another one (selected randomly again). There must be one, so you will find one earlier or later (until you have a unique puzzle).
Thus the clue, which is added, is not a random one out of all possible (in the empty cells), but only out of all which still leave a valid puzzle.

But as i showed in my simple counter-example, this way the puzzles with a fixed number of clues are not guaranteed to be found with the same probability (and i guess that is basic knowledge for mathematicians).

Back to top

Red Ed

Posted: Sun Sep 20, 2009 9:52 am Post subject:

Joined: 06 Jun 2005
Posts: 712

eleven wrote:

> **Red Ed wrote:**
>
> > Actually *eleven* is correct with his formula, though as he points out it is a totally impractical algorithm.
>
> Thanks for confirmimg that.

To be clear, it's only correct if you replace

Code:

```
if the puzzle is minimal, report it,
```

with

Code:

```
if the puzzle is minimal, report it, else print "oops"
```

... i.e. making sure there is an output (even just "oops") at every attempt. Denis was right to pull me up on that.