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THE REAL DISTRIBUTION OF MINIMAL PUZZLES

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Author

Message

denis_berthier

Posted: Mon Jul 13, 2009 8:33 am Post subject:



Joined: 19 Jun 2007
Posts: 725
Location: Paris, France

eleven wrote:

denis_berthier wrote:

secondary school level

Thats for me 😊

Quote:

step 1) all the puzzles (minimal or not) with mmax clues have the same probability of being reached by a top-down generator.
proof: as, at every step, the generator deletes one of the remaining clues with equal probabilities, for any $m \geq m_{max}$, each puzzle with m clues has probability $1/N * 1/81! * (81 - m)! * m!$ of being reached.

Please continue.

step 2) puzzles with $m_{max}-1$ clues

Forget all the minimal puzzles that have m_{max} clues. All the other puzzles with m_{max} clues have the same probability of being reached by the top down generator (as shown in step 1)

From each of them, the generator will build the same number of puzzles with $m_{max} - 1$ clues, all with the same probability.

Among these puzzles with $m_{max} - 1$ clues, we'll find all the minimal puzzles with $m_{max} - 1$ clues, which therefore all have the same probability of being reached by the generator.

Still OK?

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denis_berthier

Posted: Mon Jul 13, 2009 8:41 am Post subject:



Joined: 19 Jun 2007
Posts: 725
Location: Paris, France

coloin wrote:

Ok.....the chances of getting to the same number of clues would appear to be the same.
Before you do go on consider this 40-clue subgrid [all clues are superfluous]

Code:

```
1 . . . . 6 . . 945.78 . . . . 7.9.2354.2.4.9.37.6 . . 57.914.75 . . 1 . . . . 12.4.57 . . 2 . . . . . . 6.935421
```

The puzzles dont come out with the same frequency.

Ive confirmed it even with a larger sample.

Maybe the effect will be very small for generating from a full grid though.

My proof is only valid for puzzles issued from all the solution grids, not from a subgrid.

If you find notably different frequencies between the 20s, you must have some bias in your program.

Let there be no ambiguity: I'm not saying that all the minimal puzzles have the same chances. It depends on (and only on) their number of clues (in the specific way described in my first post).

Last edited by denis_berthier on Wed Jul 15, 2009 11:11 am; edited 2 times in total

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eleven

Posted: Mon Jul 13, 2009 8:55 am Post subject:

[quote](#)

Joined: 10 Feb 2008
Posts: 350

denis_berthier wrote:

step 2)
...
Still OK?

Yes, i think the minimals at level $n_{max}-1$ will have the same probability. Wait no, i have to think about it first.

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eleven

Posted: Mon Jul 13, 2009 9:07 am Post subject:

[quote](#)

Joined: 10 Feb 2008
Posts: 350

ok, at this level i cant see a problem.

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denis_berthier

Posted: Mon Jul 13, 2009 9:11 am Post subject:

[quote](#) [edit](#)

Joined: 19 Jun 2007
Posts: 725
Location: Paris, France

eleven wrote:

ok, at this level i cant see a problem.

Step 3) level $m_{max}-2$

Forget all the minimal puzzles that have m_{max} or $m_{max}-1$ clues.

From each of the remaining puzzles with $m_{max}-1$ clues obtained at step 2, the generator creates puzzles with $m_{max}-2$ clues, all with the same probability.

Among them we can find all the minimal puzzles with $m_{max}-2$ clues, which therefore all have the same probability.

If you're OK again, I think you are ready to do the next steps by yourself.

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eleven

Posted: Mon Jul 13, 2009 9:37 am Post subject:

[quote](#)

Joined: 10 Feb 2008
Posts: 350

No, i dont think so. After deleting subtrees there are less paths to some nodes than to others in the lower levels.

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denis_berthier

Posted: Mon Jul 13, 2009 10:30 am Post subject:

[quote](#) [edit](#)

Joined: 19 Jun 2007
Posts: 725
Location: Paris, France

eleven wrote:

No, i dont think so. After deleting subtrees there are less paths to some nodes than to others in the lower levels.

At what level would this begin? m_{max} , $m_{max}-1$, $m_{max}-2$... ?

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eleven

Posted: Mon Jul 13, 2009 11:07 am Post subject:

[quote](#)

Joined: 10 Feb 2008
Posts: 350

I made a small sample with 5 numbers to get it into my head.

So lets say, if a node is "minimal", then all nodes below which dont have another number, are minimal too.
Let "12..." be minimal.

Code:

```

12345
.2345      1.345      12.45      123.5      1234.
..345      .2.45      .23.5      .234.      1..45      1.3.5      1.34.      12..5      12.4.
123..
...45      ..3.5      ..34.      .2..5      .2.4.      .23..      1...5      1..4.      1.3..
@12...

```

Now look at the chances to reach "1.3.." If you removed 4 and 5 in the first 2 steps, they are 50%.

If you removed 2 and 5 in the first steps, they are 33%. This is not fair.

So the probability of reaching a node depends on the path.

Dont you think, thats a problem ?

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denis_berthier

Posted: Mon Jul 13, 2009 11:27 am Post subject:

[quote](#) [edit](#)

eleven wrote:

I made a small sample with 5 numbers to get it into my head.
So lets say, if a node is "minimal", then all nodes below which dont have another number, are minimal too. Let "12..." be minimal.

If a node is minimal, there won't be any minimal node below, not even any node at all: if you follow my proof, which parallels the workings of a top-down generator, they will be forgotten.

(BTW, any node below a minimal one would have multiple solutions).

In order to understand the proof, you must proceed in the order I indicated: from mmax downwards.

Please try to spot the place where you think the problem you're mentioning might first appear: mmax, mmax-1, mmax-2.

One thing you must take into consideration is that, at any level, there can't be any interaction between the puzzles still under consideration at this level and the minimal puzzles forgotten at the higher steps or any of their mumti-solution descendants.

All that is above the puzzles still under consideration when we reach level n (as we are going downwards) is unchanged by the fact that we have discarded the minimal puzzles and their multi-solution descendants at the previous levels.

In particular, the number of paths to any puzzle remaining under consideration at level n is unchanged by these eliminations.

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Red Ed

Posted: Mon Jul 13, 2009 12:13 pm Post subject:

[quote](#)

I said I'd given up trying to be helpful, so I'll limit myself to dropping just a small hint: *think about what happens at level 77.*

Joined: 06 Jun 2005
Posts: 583

Have fun pondering that. I'm off to catch a plane. 😊

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denis_berthier

Posted: Mon Jul 13, 2009 12:15 pm Post subject:

[quote](#) [edit](#)

Red Ed wrote:

I said I'd given up trying to be helpful, so I'll limit myself to dropping just a small hint: *think what happens at level 77.*

Have fun pondering that. I'm off to catch a plane. 😊

Joined: 19 Jun 2007
Posts: 725
Location: Paris, France

At level 77, nothing happens, because there's no minimal puzzle above.

Unless you mean 81-77, in which case I can't believe you don't know reasoning by recursion.

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m_b_metcalf

□ Posted: Mon Jul 13, 2009 12:39 pm Post subject:

[quote](#)

Joined: 15 May 2006
Posts: 2216
Location: Berlin

The discussion on this thread reminds me of the [Monty Hall Problem](#). Might it be a useful exercise for all participants to start out on neutral ground by agreeing to *that* problem's solution?

Regards (and just trying to be helpful),

Mike Metcalf

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denis_berthier

□ Posted: Mon Jul 13, 2009 12:45 pm Post subject:

[quote](#) [edit](#)

Joined: 19 Jun 2007
Posts: 725
Location: Paris, France

m_b_metcalf wrote:

The discussion on this thread reminds me of the Monty Hall Problem[/[url](#)]. Might it be a useful exercise for all participants to start out on neutral ground by agreeing to *that* problem's solution?

Nice puzzle.
But what we're after here is much more elementary.

Regards.

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eleven

□ Posted: Mon Jul 13, 2009 1:48 pm Post subject:

[quote](#)

Joined: 10 Feb 2008
Posts: 350

Though it was a bit irritating, that there is a different probability for reaching cells, i cant find a reason that [edit - was not clear] for a fixed level the probability to find a minimal could be different for different nodes (so i also dont trust Coloins result).

Now that i made this sample, you maybe can demonstrate your formula with it. Lets say, the 4 cells marked @ are minmal.

Code:

```

12345
.2345      1.345      12345      123.5      1234.
..345 .2.45 .23.5 .234. 1..45 1.3.5 1.34. 12..5 12.4.
123..
...45 ..3.5 ..34. .2..5 .2.4. .23.. 1...5 1..4. 1.3..
@12...
      @....5      @...4.      @..3..      #.2...      #1....
                        #.....

```

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denis_berthier

□ Posted: Mon Jul 13, 2009 3:05 pm Post subject:

[quote](#) [edit](#)

eleven wrote:

Though it was a bit irritating, that there is a different probability for reaching cells,

Joined: 19 Jun 2007
Posts: 725
Location: Paris, France

Why irritating?

Because it means that the top-down generators are biased?

As shown in my first post, this doesn't disqualify them. We just have to apply the proper correction factors.

Of course, that's a little more work for us.

There's also the possibility of filtering the generator, by giving each of the puzzles it outputs a probability of being kept or thrown away. These probabilities should be consistent with the cf-correction factors I've given. Unfortunately, this means that about only a few puzzles in 1000 could be kept.

eleven wrote:

i cant find a reason that the probability to find the minimals of the same level should be different after dropping the subtrees.

Now that i made this sample, you maybe can demonstrate your formula with it. Lets say, the 4 cells marked @ are minimal.

Code:

```

                12345
                12.45   123.5   1234.
..345   .2.45   .23.5   .234.   1..45   1.3.5   1.34.   12..5
12.4.   123..
...45   ..3.5   ..34.   .2..5   .2.4.   .23..   1...5   1..4.
1.3..   @12...
                @....5   @...4.   @..3..   #.2...   #1....

```

I'm not sure exactly what you're expecting.

I suppose you have counted all the paths to each puzzle down to line 4, where your first @ appears, and checked that all the puzzles at this floor have equal probas of being reached from the top (1/10 exactly). Then it's easy to check that all the puzzles on line 5 that are not below @12... (i.e the first 3) all have 4 parents on line 4, all of which have the same number of paths coming from the top. Therefore all the puzzles on line 5 not below @12... have the same number of paths leading to them and the same proba. (And you can see that what's below @12... is irrelevant).

As your mini example has only 5 cells instead of 81, my formula won't be valid in exactly the form I've given.

But it will become $cf(n+1)/cf(n) = (5-n)/(n+1)$ - just replace 81 by 5 - with $n=5$ for top floor.

Take $cf(5)=1$ (only 1 puzzle at floor 5).

With $n = 4$ (first line below the top), we get $cf(5)/cf(4) = 1/5$ and $cf(4) = 5$ (==> each of the puzzles at floor 4 has proba 1/5)

With $n=3$ (second line below the top), we get $cf(4)/cf(3) = 1/2$ and $cf(3) = 10$ (==> each of the puzzles at floor 3 have proba 1/10)

With $n=2$ (third line below the top), we get $cf(3)/cf(2) = 1$ and $cf(2) = 10$ (==> each of the puzzles at floor 2, including @12..., has proba 1/10)

With $n=1$ (fourth line below the top, last line), we get $cf(2)/cf(1) = 2$ and $cf(1) = 5$ (==> each of the puzzles at floor 1 which is not below @12..., i.e. @...5 , @...4. and @..3.., has proba 1/5)

Now, you can check directly that this prediction of my (obvious) theory is true.

Each of the puzzles at floor 2 has probability 1/10. It has 2 sons, each of which inherits from it half of its proba, i.e. 1/20.

But each of the puzzles at floor 1 has 4 fathers (God forgive his mother! 😊), each of which transmits the same heritage. It has therefore probability 1/5.

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