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THE REAL DISTRIBUTION OF MINIMAL PUZZLES

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Author

Message

Red Ed

Posted: Thu Oct 22, 2009 6:22 am Post subject:



Joined: 06 Jun 2005
Posts: 858

denis_berthier wrote:

Though systematically below, it is consistent with sampling errors.

"Consistently below" makes me think that the SE ratings for Allan's puzzles are subtly different to those for unbiased puzzles.
We should expect them to be, given [past experience](#).

Interesting that Allan's result for 32s is slightly above what **eleven** and I reported.

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denis_berthier

Posted: Thu Oct 22, 2009 6:33 am Post subject:



Joined: 19 Jun 2007
Posts: 970
Location: Paris, France

Red Ed wrote:

given [past experience](#).

A reference which was only a summary of my anterior reports 😊

Red Ed wrote:

Interesting that Allan's result for 32s is slightly above what **eleven** and I reported.

Interesting but, considering sampling errors, no more meaningful than the above difference.

What I'd like to see now is the #clues distribution if Allan can run his generator with no preset number of clues.

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Red Ed

📄 Posted: Thu Oct 22, 2009 7:18 am Post subject:



Joined: 06 Jun 2005
Posts: 858

Actually the point of referencing that post, which goes beyond what you had previously observed, was to suggest that a similar distributional analysis might be worthwhile with new data - *i.e.* Allan's puzzles just produced *versus* a set of puzzles produced in an unbiased fashion (both for the same fixed number of clues). It would tell us so much more than the mean & sd alone.

But if it makes your day to be reminded of your past work, then I'm happy to have obliged 😊

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Red Ed

📄 Posted: Thu Oct 22, 2009 7:33 am Post subject:



Joined: 06 Jun 2005
Posts: 858

btw, mean SER on 1400+ 31-clue puzzles produced by the supersets method is 6.94; no time to produce a measure of confidence for that now though.

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Allan Barker

📄 Posted: Thu Oct 22, 2009 4:52 pm Post subject:



Joined: 21 Feb 2008
Posts: 362
Location: Bangkok

Denis Berthier wrote:

A quick SER computation gives: ...
We can see a very strong upward trend.

Although not enough for any statistical confidence, I now have 5 37-clue puzzles with average SER of 8.0.

Denis Berthier wrote:

In the range where comparisons are possible, it is closer to the real values than any upward or top-down generator.
Though systematically below, it is consistent with sampling errors.

That's interesting. Although the routine starts by removing 81-N candidates from a grid, the grid is then randomized during the convergence process (clues change). However, there would still be bias from this remaining process.

Red Ed wrote:

"Consistently below" makes me think that the SE ratings for Allan's puzzles are subtly different to those for unbiased puzzles.

Yes, I assume that is the bias I mention.

Denis Berthier wrote:

What happens if you don't a preset the number of clues?

I don't know 🚫 If the number of clues can change in a search for fewer non-

essential clues, it could revert to a top down generator. As **Ed Red** mentioned though, comparing distributions would be interesting. I'll see if there is a way to get a distribution while retaining the essence of the method.

Last edited by Allan Barker on Sat Oct 24, 2009 1:10 am; edited 1 time in total

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Red Ed

Posted: Thu Oct 22, 2009 5:31 pm Post subject:

 [quote](#)

Joined: 06 Jun 2005
Posts: 858

Allan, for distributions, it would suffice to just produce a list of (SER,#clues) pairs; then we can cut it any way we like for distribution info - e.g. fixed number of clues (e.g. is your method more prone to fish than unbiased generation?), or fixed method (e.g. how does fishiness depend on #clues in your method?).

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eleven

Posted: Fri Oct 23, 2009 8:07 am Post subject:

 [quote](#)

Joined: 10 Feb 2008
Posts: 549

OT:

I was interested, how the $\{-n+m\}$ performs to find high clue puzzles. I took Red Eds big 32 cluster as starting point and used gsf's program.

Code:

```

30 32's: min 2, max
8.3, average 4.88
{-1+2},{-1+1} -> 9/ 41 33's: min 2.5, max
7.2, average 6.05
{-1+2},4x{-1+1} -> 5/ 66 34's: min 2.6, max 9,
average 6.99
{-1+2},{-2+2},4x{-1+1} -> 6/156 35's: min 2.6, max
9.1, average 7.91
{-1+2} -> 33 36's: min 7.1, max 9,
average 7.91
{-1+2} -> 1 37 : 9.0

```

This took about 11 hours. The $\{-1+2\}$ for the 156 35's was extremely slow (more than 5 hours, where for 6 puzzles it needed 7 minutes), so there could be done a lot of optimization.

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eleven

Posted: Fri Oct 23, 2009 8:33 am Post subject:

 [quote](#)

Joined: 10 Feb 2008
Posts: 549

In a background process i have running the supersets method for 33 clues (from 24).

The (intermediate) result for 0.5 mio random grids is

Code:

```

jct[ 0 ] = 508126
jct[ 1 ] = 37
jct[ 2 ] = 6
jct[ 3 ] = 2
jct[ 4 ] = 3
jct[ 7 ] = 1

```

```

jct[1000+] =          0

508175 grids, 74 puzzles
Estimated mean number of proper minimal 33s per grid =
8.704923e+08
Estimated relative error =
17.93%

74 puzzles, SER min 2, max 8.8, average 7.26

```

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Allan Barker

Posted: Fri Oct 23, 2009 10:29 am Post subject:



Joined: 21 Feb 2008
Posts: 362
Location: Bangkok

Red Ed wrote:

Allan, for distributions, it would suffice to just produce a list of (SER,#clues) pairs; then we can cut it any way we like for distribution info - e.g. fixed number of clues (e.g. is your method more prone to fish than unbiased generation?), or fixed method (e.g. how does fishiness depend on #clues in your method?).

I don't think there is anything more fishy 😊 about this algorithm vs. any other (not including true unbiased generation). It treats the grid as a 9x9x9 cube and randomly chooses x,y,z pairs for swapping. The only decision is then whether the grid is valid, minimal, or non-minimal, which is necessary even for unbiased generation.

Where the bias would creep in, and I'm sure it does, is the restriction to move one pair at a time. Now imagine choosing the number of pairs to swap each time, 1, 2, ... N, when N = the number of clues. When you change all N clues each time, you arrive at a completely unbiased generator (and its performance). If worthwhile, I could make a few data sets where the algorithm arrives at a minimal by changing different numbers of clues each step. You might then be able to measure the change in bias from one set to the next with your 3222 patterns test (?)

Eleven, I am wondering how similar this is to your {+n,-m} method. Is there a good link that explains that method in detail?

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eleven

Posted: Fri Oct 23, 2009 11:47 am Post subject:



Allan Barker wrote:

Eleven, I am wondering how similar this is to your {+n,-m} method. Is there a good link that explains that method in detail?

No link needed, {-n+m} means, remove n clues and add m. For a puzzle with N givens you have $\text{choose}(N,n) \cdot \text{choose}(81-N+n, m) \cdot 9^m$ possibilities to find minimals.

I denoted the post as OT, because i had seen before, that this method produces rather strong bias.

[Edit:]oops, i forgot the 9^m factor, the point is, that it is not restricted to a fixed grid.

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Posted: Sat Oct 24, 2009 5:54 am Post subject:



Joined: 19 Jun 2007
 Posts: 970
 Location: Paris, France

ANALYSIS OF SKEWNESS AND KURTOSIS

Until now, I have given results for the mean, standard-deviation and distribution for various random variables of minimal puzzles: number of clues, SER, NRCZT.

But there are two more variable of interest: skewness and kurtosis.

Skewness is a measure of the asymmetry of the distribution

- negative skew: the left tail is longer; the mass of the distribution is concentrated on the right of the figure; the distribution has relatively few low values.
- positive skew: the right tail is longer; the mass of the distribution is concentrated on the left of the figure; the distribution has relatively few high values.

Details on Wikipedia (<http://en.wikipedia.org/wiki/Skewness>)

Kurtosis (also called "excess kurtosis" to avoid ambiguities) is a measure of how much the distribution shape differs from that of a Normal distribution with the same mean and standard deviation as X.

- lower kurtosis means more of the variance is due to frequent modestly sized deviations;
- higher kurtosis means more of the variance is due to infrequent extreme deviations.

Kurtosis can vary between - 2 and + infinity.

Details on Wikipedia (<http://en.wikipedia.org/wiki/Kurtosis>)

1) number-of-clues***Kurtosis***

bottom-up generator (suexg1.4) : 0.026
 top-down generator (suexg-x.x) : 0.007 (Allan's top-down = 0.014)
 controlled-bias generator (gsf | suexg-cb) : 0.023 (suexg | .suexg-cb : 0.013)

Skweness

bottom-up generator (suexg1.4) : 0.11
 top-down generator (suexg-x.x) : 0.08 (Allan's top-down = 0.11)
 controlled-bias generator (gsf | suexg-cb) : 0.08 (suexg | .suexg-cb : 0.08)

For all these generators, kurtosis and skewness of the number of clues are ~ 0 and the general shape of the number-of-clues distribution looks much like Normal.

2) SER

Kurtosis

bottom-up generator (suexg1.4) : -1.08
 top-down generator (suexg-x.x) -1.36 : (Allan's top-down = -1.35)
 controlled-bias generator (gsf | suexg-cb) : - 1.70 (suexg | suexg-cb : -1.70)

Skweness

bottom-up generator (suexg1.4) : 0.79
 top-down generator (suexg-x.x) : 0.60 (Allan's top-down = 0.60)
 controlled-bias generator (gsf | suexg-cb) : 0.21 (suexg | suexg-cb : 0.21)

3) NRCZT**Kurtosis**

bottom-up generator (suexg1.4) : 0.91
 top-down generator (suexg-x.x) : 0.27 (Allan's top-down = 0.34)
 controlled-bias generator (suexg | .suexg-cb) : -0.46

Skweness

bottom-up generator (suexg1.4) : 1.24
 top-down generator (suexg-x.x) : 1.01 (Allan's top-down = 1.01)
 controlled-bias generator (suexg | .suexg-cb :) 0.65

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denis_berthier

Posted: Sat Oct 24, 2009 5:55 am Post subject:

 [quote](#)  [edit](#)

Joined: 19 Jun 2007
 Posts: 970
 Location: Paris, France

KURTOSIS PROFILES

For a random variable X other than the number of clues, kurtosis(X) can be computed for the subsamples of n-clue puzzles.
 We thus get a "kurtosis profile".

Kurtosis-profile(SER)

As shown by the tables below, all the generators we have studied have similar kurtosis profiles :

- negative values in the range [22, 28]
- a negative peak around 26/27 clues (notice: around the real mean number of clues and not around the mean number of clues for the generator)
- positive values above 30 (when there are puzzles there)

Moreover, the controlled-bias generator has exactly the same kurtosis profile, whether the source of complete grids is suexg or gsf :

Code:

```
controlled-bias generator (gsf | suexg-cb) :
21      -0.65 (*)
22      -0.24
```

```
23      -0.72
24      -1.18
25      -1.53
26      -1.73
27      -1.68
28      -1.31
29      -0.32
30      2.62 (*)
31      -0.64 (*)
```

Code:

```
controlled-bias generator (suexg | suexg-cb) :
21      -1.5 (*)
22      -0.48
23      -0.73
24      -1.18
25      -1.54
26      -1.73
27      -1.67
28      -1.29
29      -0.48
30      1.23 (*)
```

Code:

```
top-down generator (suexgx.x) :
21      0.29 (*)
22      -0.50
23      -0.88
24      -1.21
25      -1.48
26      -1.68
27      -1.74
28      -1.59
29      -1.28
30      3.64 (*)
```

Code:

```
bottom-up generator (suexgl.4) :
20      0.25
21      0.28
22      -0.28
23      -0.69
24      -1.08
25      -1.41
26      -1.66
27      -1.74
28      -1.54
29      1.57 (*)
```

Allan's montecarlo is interesting because it shows how the kurtosis evolves beyond 30 clues: there is a continuous increase, i.e. values away from the mean tend to be more and more frequent.

The skewness profile will show that these values away from the mean tend to be below the mean.

Code:

```
Allan's montecarlo :
22      -0.36
23      -0.41
24      -1.18
25      -1.55
26      -1.71
27      -1.72
28      -1.44
29       0.56
30       0.26
31       1.93
32       2.88
33       5.38
34       5.90
35       8.48
36       4.37 (*)
```

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denis_berthier

Posted: Sat Oct 24, 2009 5:55 am Post subject:



Joined: 19 Jun 2007

Posts: 970

Location: Paris, France

SKEWNESS PROFILES

For a random variable X other than the number of clues, skewness(X) can be computed for the subsamples of n-clue puzzles.

We thus get a "skewness profile".

Skewness-profile(SER)

Code:

```
controlled-bias generator (gsf | suexg-cb) :
21      2.38 (*)
22      0.001
23      0.018
24      0.086
25      0.14
26      0.045
27     -0.037
28     -0.024
29     -0.0056
30     -0.0006
31     -1.3e-5 (*)
There's almost no skewness
```


Code:

```
top-down generator (suegx.x) :
21      0.0003 (*)
22      0.037
23      0.15
24      0.24
25      0.15
26      0.03
27     -0.0006
28     -0.0009
29     -0.0001
30     -2e-5 (*)
```

Code:

```
bottom-up generator (suegx1.4) :
20      0.0004
21      0.012
22      0.095
23      0.27
24      0.29
25      0.12
26      0.016
27     -6e-6
28     -0.0002
29     -4e-5 (*)
```

Although its skewness profile is different from all the others, Allan's montecarlo is interesting, because it can be computed upto 36 clues and it shows how puzzles with many clues (higher mean SER, higher kurtosis) tend to avoid having much higher complexity than the mean SER value for their number of clues: there is a decreasing skewness, i.e. the SER of puzzles with more clues tend to be below the mean SER of their number of clues.

Code:

```
Allan's montecarlo :
22      1.12
23      1.11
24      0.74
25      0.44
26      0.20
27     -0.11
28     -0.51
```

| | |
|----|-----------|
| 29 | -0.99 |
| 30 | -1.27 |
| 31 | -1.68 |
| 32 | -1.85 |
| 33 | -2.22 |
| 34 | -2.32 |
| 35 | -2.45 |
| 36 | -1.63 (*) |

It is as if there was a barrier of complexity and, when the number of clues increases:

- the mean complexity increases
 - the proportion of puzzles away from the mean increases
- but
- there are more and more puzzles far below this mean
 - there are fewer and fewer puzzles far above this mean

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Red Ed

Posted: Sat Oct 24, 2009 7:53 am Post subject:

 [quote](#)

Joined: 06 Jun 2005
Posts: 858

denis_berthier wrote:

For all these generators, kurtosis and skewness of the number of clues are ~ 0 and the general shape of the number-of-clues distribution looks much like Normal.

Nice. If I find the time this weekend then I'll redo my analysis of the fit of the Gamma distribution to this family. The analysis was done originally on "suexg | .suexg-cb". Recall that after proposing Normal as a good approximation, I offered that Gamma might be a better fit: and for the parameters that I proposed, the skewness works out at 0.087 and the kurtosis at 0.011 -- good matches for your 0.08 and 0.013 respectively.

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denis_berthier

Posted: Sat Oct 24, 2009 8:40 am Post subject:

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Joined: 19 Jun 2007
Posts: 970
Location: Paris, France

Red Ed wrote:

If I find the time this weekend then I'll redo my analysis of the fit of the Gamma distribution to this family.

If you can wait a few more days, I'll soon have a larger sample.

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