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### THE REAL DISTRIBUTION OF MINIMAL PUZZLES

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#### Author

#### Message

**coloin**

Posted: Sat Jul 18, 2009 10:57 pm Post subject:



Joined: 06 May 2005  
Posts: 1052  
Location: Devon UK

#### denis\_berthier wrote:

```
P41 = P40
P39 = 42/40 * P40
P38 = 43/39 * P39
```

Thank-you **denis** now I understand what the formula means [obvious but not to everyone] The  $P_{41}=P_{40}$  is very nice.

The hike up from 25.39 to ~26.5 is because of

#### Code:

```
unbiased-mean(X) = sum[E(X, n) * on(n) * cf(n)] /
sum[on(n) * cf(n)].
```

except I don't get this bit apart from I think it is related to the statistical bias introduced by the original bias.

#### Allan Barker wrote:

```
39 ?????? Were they using specially selected grids or random ones?
```

Initially I started out trying to find large puzzles in specific grids - but the 37s came out of neighbouring grids. The 38s and 39s were found probably by processing power [with the help of IBM and 16-core super machines I believe !]

An analytical approach such as yours to reproduce this work would be commendable. Analysis on the individual essential clues and non essential clues in a subgrid might point the way to go in achieving the progressively harder step-up in clues. It might indicate which puzzle is more likely to have a bigger brother. I wish you luck !

C

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**Red Ed**

Posted: Sun Jul 19, 2009 6:45 am Post subject:



Joined: 06 Jun 2005  
Posts: 616

Coloin, with regard to the bit you didn't get ... if  $E(X,n)$  is the average value of  $X$  (any thing "X") over  $n$ -clue minimals, and  $P(n)$  is the probability that a random minimal has  $n$  clues, then  $\sum_{n} \{ E(X,n) * P(n) \}$  is the average value of  $X$

over all minimals. Just a weighted average, right?

Reproduced below in small font is how  $P(n)$  is derived. This has been said many times before, but it wasn't clear to me if you understood or not.

**Quote:**

I think about it this way:

1. The modified generator finds  $on(n)$   $n$ -clue minimals in  $t$  trials: so estimated probability  $on(n)/t$  that a random  $n$ -clue subgrid is a proper minimal puzzle.
2. There are  $choose(81,n)$   $n$ -clue subgrids in any solution grid.
3.  $1\&2 \Rightarrow$  estimated  $nr$   $n$ -clue minimals per grid, call it  $EM(n)$ , equals  $on(n)/t * choose(81,n)$ .
4. Therefore the estimated total number of minimals per grid, call it  $EM(*)$ , equals  $EM(0)+EM(1)+...+EM(81)$
5. ... and the estimated proportion of  $n$ -clue minimals, call it  $P(n)$ , equals  $EM(n)/EM(*)$ .

If we write out  $P(n)$  in full we get:

$$P(n) = on(n)/t * choose(81,n) / \sum_{c=0...81} ( on(c)/t * choose(81,c) )$$

You can multiply top and bottom by the same number and the formula will remain true, obviously. Denis chooses to multiply by  $t/choose(81,24)$  and he uses the name  $cf(n)$  to mean  $choose(81,n)/choose(81,24)$ . So, in Denis' terms:

$$P(n) = on(n)*cf(n) / \sum_{c=0...81} ( on(c)*cf(c) )$$

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**m\_b\_metcalf**

Posted: Sun Jul 19, 2009 9:50 am Post subject:



Joined: 15 May 2006  
Posts: 2218  
Location: Berlin

I'm back for a few hours and see much has taken place. This is just to point out that I have a 24MB zipped file of 1 million random grids that I'd be happy to make available if anyone wants them and can tell me how to transmit them.

Regards,

Mike Metcalf

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**denis\_berthier**

Posted: Mon Jul 20, 2009 7:20 am Post subject:



Joined: 19 Jun 2007  
Posts: 752  
Location: Paris, France

**denis\_berthier wrote:**

**Allan Barker wrote:**

**denis\_berthier wrote:**

**Allan Barker wrote:**

If anyone is interested, I can put the first batch of 30s on my website.

I'm interested. Large samples of 30s, 31s, 32s ... are not so frequent.

<http://sudokuone.com/xsudo1/puzzle33.txt>. These are

not compressed, a bit over 20000.

Thanks, I don't know when I can study them (all my computing resources are currently centered on the controlled-bias generator), but I'll do it.

BTW, the url is: <http://sudokuone.com/xsудо1/puzzle30.txt>

I found a little time to run SudokuExplainer on your collection of 20,000 30s. The mean value and standard deviation for the SER are:

$E(\text{SER}) = 6.255$

$sd(\text{SER}) = 2.124$

This is fully consistent with the results obtained previously on 2 much smaller samples:

- the 10 30-clue puzzles included in sudogen0\_1M

$E(\text{SER}) = 6.46$

$sd(\text{SER}) = 1.74$

- the 8 30-clue puzzles included in rabrnd\_1M

$E(\text{SER}) = 5.69$

$sd(\text{SER}) = 2.02$

and with their weighted average:

$E(\text{SER}) = 6.07$

$sd(\text{SER}) = 1.96$

This result was uncertain due to the small sample size.

Now, with your large sample, it is confirmed that the mean SER increases with the number of clues in a range that includes the 30s.

Did you say you have large samples of 31s, 32s and 33s?

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**Allan Barker**

Posted: Mon Jul 20, 2009 11:50 am Post subject:

 [quote](#)

**denis\_berthier wrote:**

Did you say you have large samples of 31s, 32s and 33s?

Joined: 21 Feb 2008

Posts: 295

Location: Bangkok

Some but not much yet as I have spent more time tinkering with the code. I do have about **850** size 33s, which would be interesting to test. These were generated without the constraint of 1 minimal per grid, i.e., the grid is free to change during the search. I do plan to make more including 32s and 31s soon.

From now on, I will paste data files on a separate webpage for data <http://sudokuone.com/xsудо1/data.htm>. The files are there now.

BTW: Thanks, I corrected the erroneous file name.

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**denis\_berthier**

Posted: Mon Jul 20, 2009 12:15 pm Post subject:

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**Allan Barker wrote:**

I do have about **850** size 33s, which would be interesting to test.

Joined: 19 Jun 2007

Posts: 752

Posts: 752  
Location: Paris, France

Thanks. I'll try to test them today.

**Allan Barker wrote:**

These were generated without the constraint of 1 minimal per grid, i.e., the grid is free to change during the search.

Does it mean anything special wrt to the problem of bias? Did you implement some form of the controlled bias?

**Allan Barker wrote:**

I do plan to make more including 32s and 31s soon.

Even only ~ 1000 of each would be enough to check whether the trend extends to the whole range 20-33.

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**denis\_berthier**

Posted: Mon Jul 20, 2009 1:16 pm Post subject:



**Allan,**

Joined: 19 Jun 2007  
Posts: 752  
Location: Paris, France

For the 850 33s:

mean SER = 7.14  
standard deviation = 1.47

Here again, the trend is confirmed.

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**denis\_berthier**

Posted: Tue Jul 21, 2009 4:53 am Post subject:



Joined: 19 Jun 2007  
Posts: 752  
Location: Paris, France

**NEW RESULTS WITH THE CONTROLLED-BIAS GENERATOR**

The controlled-bias generator is slow but it works fine.  
I've now generated 10,000 minimal puzzles.

As suexg was free software, the same is true of its controlled-bias modification (implemented by eleven): suexg-cb; it can be downloaded from my web page on classification: <http://www.carva.org/denis.berthier/HLS/Classification>.

I've also improved the redaction of section 3 on the controlled-bias generator and its application to the unbiased classification of minimal puzzles.

Here are the mean and standard deviations obtained from these 10,000 puzzles for a few variables of interest. These results are very stable (almost unchanged when the sample passed from 1,000 to 10,000 puzzles).

Number of clues of minimal puzzles:  
unbiased-mean = 26.568

SER  
unbiased-mean = 4.484 unbiased-sd = 2.53

NRCZT  
unbiased-mean = 2.31 unbiased-sd = 1.38

As could be expected, there are more fluctuations in the distribution of clues, especially in the tail, but it is nevertheless reasonably stable.

For comparison with the standard, non-controlled, top-down generator, the second column ("top-down") recalls the results obtained from the 1,000,000 puzzles generated with suexg-x.x; the third column ("controlled") gives the direct result for the sample of 10,000 puzzles from the controlled-bias generator; the fourth column ("unbiased") gives the unbiased results, obtained by using the correction factors, it is scaled to 1,000,000 puzzles.

The mean values for each case are also recalled.

Of course, "top-down" appears to be much more biased in favour of fewer clues than "controlled".

**Code:**

```
#clues  top-down  controlled  unbiased
19      0         0           0.0 (*)
20     44         0           0.0 (*)
21    2428         1           1.14 (*)
22   34548        11          34.34 (*)
23  172512       188         1505.35
24  342335      1211        23433.75
25  297838      3066        135270.99
26  122116      3372        320431.17
27  25315       1676        324429.35
28   2686        423         157914.58
29   168         49          33431.50
30    10         3           3547.83 (*)
31     0         0           0.0 (*)

mean    24.38         25.658        26.568
```

\* values relying on a small sample should be taken with caution.

Last edited by denis\_berthier on Tue Jul 21, 2009 3:03 pm; edited 1 time in total

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**Red Ed**

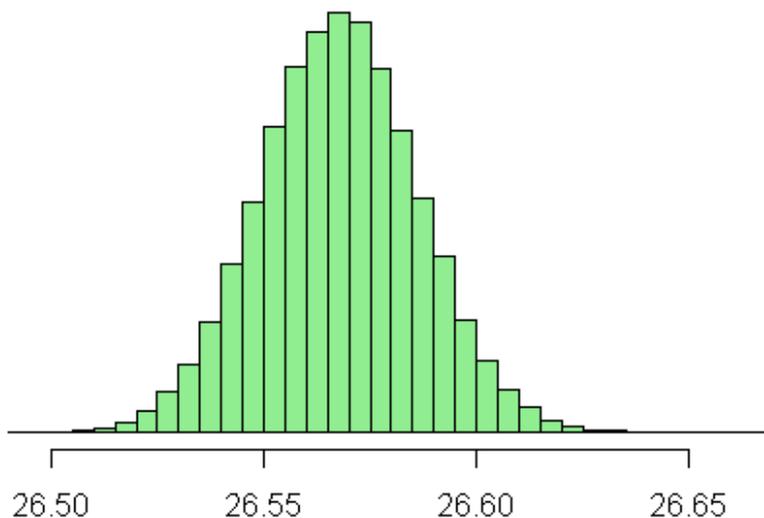
Posted: Tue Jul 21, 2009 2:19 pm Post subject:

[quote](#)

Joined: 06 Jun 2005  
Posts: 616

Three 30s, purely at random! Impressive amount of compute time required for that, I should think.

Bootstrap resampling of those counts gives the following probability distribution for the average-number-of-clues value that you'd get if you ran your experiment for an enormously long time:



Of course the true average-number-of-clues might be outside of that range (after all, we already have strong evidence that the *suexg* solution grid generator produces biased number-of-minimals estimates). I ought to burn some CPU to see if I can come up with enough data to produce a similar graph, so that we can compare.

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**Red Ed**

Posted: Tue Jul 21, 2009 2:23 pm Post subject:



PS: this is a typo -

**denis\_berthier wrote:**

Number of clues of minimal puzzles:  
unbiased-mean = 26.658

You mean 26.568.

Joined: 06 Jun 2005  
Posts: 616

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**denis\_berthier**

Posted: Tue Jul 21, 2009 3:03 pm Post subject:



**Red Ed wrote:**

after all, we already have strong evidence that the *suexg* solution grid generator produces biased number-of-minimals estimates

Joined: 19 Jun 2007  
Posts: 752  
Location: Paris, France

Obviously, we don't have the same notion of "evidence".

The only evidence we have is that the *suexg* generator of minimal puzzles is biased - that's not a scoop. We have no evidence that its complete grids generator part is biased.

When asked why this imagined bias in the complete grids would lead to the same results for minimal puzzles generated by *suexg* and by Allan's generator (based on completely different principles), as described on my website, your only answer was that you had overlooked this.

Much of what is in the complete grids is irrelevant for the minimal puzzles, as I've already shown for your 3322 tests, which you once pushed forward as an argument against the *sudogen0\_1M* collection.

Moreover, the controlled-bias version of suexg, suexg-cb, uses so many more complete grids before it gets a minimal puzzle that any bias in the complete grids is very unlikely to have any effect on the minimal puzzles.

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**coloin**

Posted: Tue Jul 21, 2009 4:05 pm Post subject:



Joined: 06 May 2005  
Posts: 1052  
Location: Devon UK

Well, suexg tends to produce puzzles->grids with less U4s [10.67] as opposed to the expected 11.58.

As we know the grid does make a difference - here is data from the past with 1M puzzles per grid with suexg:

**Code:**

Ran6	Coloin37	Havard37	MC
PT	SF [2-U4s]		
24.5	24.83	24.80	25.71
25.56	24.10		
18 0	18 0	18 0	18 0
18 0	18 0	18 0	18 0
19 0	19 0	19 0	19 0
19 0	19 4.3		
20 53	20 8	20 8	20 5
20 0	20 182		
21 2281	21 409	21 489	21 56
21 9	21 6051		
22 33020	22 11093	22 12231	22 1797
22 1051	22 61826		
23 169240	23 93587	23 97379	23 21631
23 22174	23 227480		
24 340913	24 281228	24 284695	24 116439
24 137344	24 352289		
25 299993	25 351640	25 348769	25 287167
25 323361	25 248568		
26 124899	26 198947	26 195332	26 330392
26 324568	26 86061		
27 26439	27 54898	27 53222	27 184541
27 152286	27 15908		
28 2966	28 7571	28 7338	28 50751
28 35032	28 1547		
29 190	29 587	29 518	39 6735
29 3931	29 74		
30 6	30 31	30 19	30 466
30 239	30 8.6		
31 0	31 0	31 0	31 20
31 5	31 0		
32 0	32 1	32 0	32 0
32 0	32 0		

From a few random grids from suexg.....

**Code:**

```
24.35
24.51
24.56
24.41
24.31
24.40
24.42
24.55
24.42
24.30
24.38
```

```

24.51
24.38
24.50
24.49
24.44
24.38
24.33
24.47
24.51.....

24.43 average

```

One thing I have learnt from you **denis** is that a random sample from a biased sample is still biased - however thin the sampling 😊

C

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**denis\_berthier**

Posted: Tue Jul 21, 2009 4:55 pm Post subject:



Joined: 19 Jun 2007  
 Posts: 752  
 Location: Paris, France

**coloin wrote:**

Well, suexg tends to produce puzzles->grids with less U4s [10.67] as opposed to the expected 11.58.

- 1) where does the "expected" 11.58 come from? Red Ed? But what guarantee do you have that his generator of grids is unbiased? He keeps claiming it? But who has ever proven it? Worse, who has ever seen this generator or any collection of puzzles produced by it?
- 2) who said that solution grids of unbiased collections of puzzles should be unbiased?
- 3) as I said in my previous post, I have shown that the U4 count (as well as all the 3322 tests defined by Red Ed) is totally irrelevant wrt to the complexity of puzzles (my main interest here): the correlation coefficients are almost null. Some people may "earn their living" by finding bias everywhere, that doesn't make the bias exist for good in the puzzles. 😊
- 4) the experience gained from a detailed study of a few complete grids and their minimal sub-puzzles can be very misleading. I'd never care to generate 1,000,000 minimals from a single grid.
- 5) I'm still expecting explanations why suexg and Allan's generator (which differ only by the way they generate complete grids) give the same classification results for puzzles. As long as there's no answer to this question, all the claims about bias in the puzzles due to bias in the complete grids are vacuous words. The example of these two generators is strong evidence that the way the complete grids are generated, provided that it is sufficiently random, is irrelevant to the puzzles. And this is easily understandable, as 2 thirds of the complete grid has to be eliminated to get a puzzle.
- 6) finally, all this is nitpicking. The important thing here is the new notion of a controlled-bias generator. When we use it, we know where the source of bias is and we know how to correct it.

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**Red Ed**

Posted: Tue Jul 21, 2009 5:26 pm Post subject:



Joined: 06 Jun 2005  
Posts: 616

Denis, Denis. What *are* we going to do with you? 😊

I'm not going to get into an argument about the effect of generator "bias" (of any form) on puzzle "complexity" (in any sense). I really don't care.

My only interest is in making sure that our analysis of the number-of-clues-in-minimals distribution (and related problems) is accurate. When you give estimates to 5 significant figures without quantifying the variance then you being either lazy or deliberately misleading. My beautiful -- even if I do say so myself -- graph above was an attempt to *help you* quantify the variance. I thought maybe you might be grateful. Ah, well, you do like to play hard to get, don't you Denis. 😊

And as for this:

**denis\_berthier wrote:****Red Ed wrote:**

after all, we already have strong evidence that the *suexg* solution grid generator produces biased number-of-minimals estimates

Obviously, we don't have the same notion of "evidence". The only evidence we have is that the suexg generator of minimal puzzles is biased - that's not a scoop. We have no evidence that its complete grids generator part is biased.

Yes, we have [evidence](#). Why don't we take a vote? **Hey everyone, vote here:** do you think that the complete grids generator part of suexg is **biased** or **unbiased**? If the consensus is with unbiased then I will endeavour to explain myself better so that you can see where you're wrong! 😊

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**denis\_berthier**

Posted: Tue Jul 21, 2009 5:36 pm Post subject:



Joined: 19 Jun 2007  
Posts: 752  
Location: Paris, France

**Red Ed wrote:**

Why don't we take a vote? **Hey everyone, vote here:** do you think that the complete grids generator part of suexg is **biased** or **unbiased**? If the consensus is with unbiased then I will endeavour to explain myself better so that you can see where you're wrong! 😊

Red Ed, is that your notion of mathematical truth? Hey everyone, vote here: do you think that  $2+2=4$  or  $5$ ?

As usual, you are not at all trying to help, as you keep repeating, but to generate confusion.

I don't care about the bias of complete grids. I care only about a possible (and anyway small) bias in the complexity of puzzles.

Be serious and answer my question about suexg and Allan's generator.

BTW, thanks for your graph illustrating my results.

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